

Extended Slow-Roll Conditions and Primordial Fluctuations: Multiple Scalar Fields and Generalized Gravity

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Abstract. As an extension of our previous study, we derive slow-roll conditions for multiple scalar fields which are non-minimally coupled with gravity and for generalized gravity theories of the form $f(\phi, R)$. We provide simple formulae of the spectral indices of scalar/tensor perturbations in terms of the slow-roll parameters.

1. Introduction

Inflation provides not only for a compelling explanation for the homogeneity and isotropy of the universe but also for the observed spectrum of density perturbations [1, 2, 3]. In its simplest realization, slow-roll inflation predicts an almost scale invariant spectrum of density perturbations and an almost scale invariant spectrum of gravitational waves. Since the observed scalar spectral index is already close to unity [4], the slow-roll approximation is useful approximation to the reality. Then, in confrontation with observational data, it is useful to widen the range of models of slow-roll inflation to connect the observational data with microphysics models of inflation.

In our previous paper [5], we have derived the slow-roll conditions for a single scalar field which non-minimally couples to gravity and provided the formulae of the scalar/tensor spectral indices in terms of the slow-roll parameters. In this paper, extending our previous study, we derive the slow-roll conditions for non-minimally coupled multiple scalar fields and provide simple formulae of the spectral indices in terms of the slow-roll parameters. This is natural extension in light of microphysics models of inflation because the existence of many light scalars is predicted in high energy theories like supergravity or superstring. We also derive the slow-roll conditions for generalized gravity models and give the formulae of spectral indices.

In Sec.2, we derive the slow-roll conditions for multiple-field models of inflation. We give simple formulae of spectral indices and compare them with those in the Einstein frame. We give an extended assisted inflation model as an example. In Sec.3, we consider generalized gravity models of the form $f(\phi, R)$ and find that to the first order in the slow-roll approximations, the system is reduced to a single field with non-minimal coupling, to which our previous results [5] can be applied. In Appendix, we give several formulae for multiple scalar fields and for generalized gravity models in the Einstein frame which are useful for calculations in the text.

2. Slow-Roll Inflation with a Non-minimally Coupled Multiple Scalar Fields

We consider the multi-field model of inflation which couples non-minimally to gravity. The action in the Jordan frame metric $g_{\mu\nu}$ is

$$S = \int d^4x \sqrt{-g} \left[\frac{\Omega(\phi)}{2\kappa^2} R - \frac{1}{2} h_{ab} g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi) \right], \quad (1)$$

where greek indices $\mu, \nu, \dots = 0, 1, 2, 3$ denote spacetime indices; latin indices $a, b, c, \dots = 1, 2, \dots, n$ serve to label the n scalar fields and h_{ab} is a metric on the scalar field space. $\kappa^2 \equiv 8\pi G$ is the bare gravitational constant and $\Omega(\phi)R$ term corresponds to the non-minimal coupling of the scalar fields to gravity. We assume that the scalar field metric h_{ab} is not degenerate. An interesting exception is a generalized gravity of the form $f(\phi, R)$ the case which will be considered in Sec.3.

We assume that the universe is described by the flat, homogeneous, and isotropic universe model with the scale factor a . The field equations are then given by

$$H^2\Omega + H\dot{\Omega} = \frac{\kappa^2}{3} \left(\frac{1}{2}\dot{\phi}^a\dot{\phi}_a + V \right), \quad (2)$$

$$\ddot{\Omega} - H\dot{\Omega} + 2\dot{H}\Omega = -\kappa^2\dot{\phi}^a\dot{\phi}_a, \quad (3)$$

$$\frac{D}{dt}\dot{\phi}^a + 3H\dot{\phi}^a + V^{,a} - \frac{3\Omega^{,a}}{\kappa^2}(\dot{H} + 2H^2) = 0, \quad (4)$$

where the dot denotes the derivative with respect to the cosmic time, $(D/dt)\dot{\phi}^a = \dot{\phi}^b\nabla_b\dot{\phi}^a = \ddot{\phi}^a + \Gamma_{bc}^a\dot{\phi}^b\dot{\phi}^c$, ∇_a is the covariant derivative associated with h_{ab} and $V^{,a} = h^{ab}\partial V/\partial\phi^b$.

2.1. Slow-Roll Conditions

Under the slow-roll approximations, the time scale of the motion of the scalar fields is assumed to be much larger than the cosmic time scale H^{-1} . As an extended slow-rolling of the scalar fields, we assume that $\dot{\phi}^a\dot{\phi}_a \ll V$, $|\dot{\Omega}| \ll H\Omega$, $|(D/dt)\dot{\phi}^a| \ll H|\dot{\phi}^a|$, $|(D/dt)\dot{\phi}^a| \ll |V^{,a}|$, then we obtain

$$H^2\Omega \simeq \frac{\kappa^2}{3}V, \quad (5)$$

$$3H\dot{\phi}^a \simeq -V^{,a} + 6\frac{\Omega^{,a}}{\kappa^2}H^2 \simeq -\Omega^2 \left(\frac{V}{\Omega^2} \right)^{,a} := -V_{eff}^{,a}, \quad (6)$$

where in the second equation, we have assumed $|\dot{H}/H^2| \ll 1$ which should be checked later.

In the following, we derive the consistency conditions for the extended slow-rolling of the scalar field (the extended slow-roll conditions). By computing $(D/dt)\dot{\phi}$ from Eq. (6), we obtain

$$\frac{(D/dt)\dot{\phi}^a}{H\dot{\phi}^a} \simeq -\frac{\dot{H}}{H^2} - \frac{\dot{\phi}^b\nabla_b\nabla^a V_{eff}}{3H^2\dot{\phi}^a} \simeq -\frac{\dot{H}}{H^2} - \frac{\Omega\nabla_b\nabla^a V_{eff}}{\kappa^2 V} \frac{V_{eff}^{,b}}{V_{eff}^{,a}}. \quad (7)$$

Also, we note

$$\frac{(D/dt)\dot{\phi}^a}{V^{,a}} \simeq -\frac{(D/dt)\dot{\phi}^a}{3H\dot{\phi}^b} \frac{V_{eff}^{,b}}{V^{,a}}. \quad (8)$$

Moreover, from Eqs. (5) and (6),

$$\frac{\dot{\phi}^a\dot{\phi}_a}{V} \simeq \frac{\Omega V_{eff}^{,a} V_{eff,a}}{3\kappa^2 V^2}, \quad (9)$$

$$\frac{\dot{\Omega}}{H\Omega} \simeq -\frac{\Omega_{,a} V_{eff}^{,a}}{\kappa^2 V}. \quad (10)$$

Hence, we finally introduce the following three slow-roll parameters and obtain the extended slow-roll conditions:

$$\epsilon := \frac{\Omega V_{eff,a} V_{eff}^{,a}}{2\kappa^2 V^2}; \quad \epsilon \ll 1, \quad (11)$$

$$\eta_{ab} := \frac{\Omega \nabla_a \nabla_b V_{eff}}{\kappa^2 V}; \quad |\eta_a{}^b| \ll 1, \quad (12)$$

$$\delta := \frac{\Omega_{,a} V_{eff}^{'a}}{\kappa^2 V}; \quad |\delta| \ll 1, \quad (13)$$

and one subsidiary condition:

$$\left| \frac{V_{eff}^{'a}}{V_{,b}} \right| = \mathcal{O}(1). \quad (14)$$

Note that since from Eq. (5) \dot{H}/H^2 is approximated as

$$\frac{\dot{H}}{H^2} \simeq -\frac{3\dot{\phi}^a \dot{\phi}_a}{2V} + \frac{\dot{\Omega}}{2H\Omega} = -\epsilon - \frac{1}{2}\delta. \quad (15)$$

Thus, $|\dot{H}/H^2| \ll 1$ is guaranteed by these slow-roll conditions.

To sum up, the extended slow-roll conditions consist of three main conditions Eqs. (11), (12), (13) and one subsidiary condition Eq. (14).[‡]

2.2. Perturbations

In [5, 6], it is shown that the gauge invariant curvature perturbation \mathcal{R} is invariant under the conformal transformation into the Einstein frame. Then the power spectrum $P_{\mathcal{R}}(k)$ in the Jordan frame is given by that in the Einstein frame [7, 8, 9],

$$P_{\mathcal{R}}(k) = \left(\frac{\hat{H}}{2\pi} \right)^2 \hat{h}^{ab} \hat{N}_{,a} \hat{N}_{,b} = \left(\frac{H}{2\pi} \right)^2 N_{,a} N^{,a}, \quad (16)$$

where k is a comoving wavenumber at the horizon exit ($k = aH$) and the hatted variables are those in the Einstein frame defined in Appendix A and we have used $\hat{H} \simeq H/\sqrt{\Omega}$ and $\hat{h}^{ab} \simeq \Omega h^{ab}$ in the slow-roll approximation. $N(\phi)$ is the e-folding number defined by

$$N(\phi) = \int_{t(\phi)}^{t_e} H dt, \quad (17)$$

where t_e is the time at the end of inflation. N is conformally invariant in the slow-roll approximation. The following useful relation is immediately obtained

$$H = -N_{,a} \dot{\phi}^a \simeq N_{,a} \frac{V_{eff}^{'a}}{3H}, \quad (18)$$

where the slow-roll equation Eq.(6) is used in the last equality. Eq.(18) can be formally solved in terms of $N_{,a}$ as

$$N_{,a} = \frac{\kappa^2 V V_{eff,a}}{\Omega V_{eff,c} V_{eff}^{'c}} + \perp_a, \quad (19)$$

where \perp_a is a term orthogonal to $V_{eff,a}$. Since $d \ln k = d \ln aH \simeq H dt$, to the first order in the slow-roll parameters, the spectral index of scalar perturbation is then given by

$$n_S - 1 \equiv \frac{d \ln P_{\mathcal{R}}}{d \ln k} = 2 \frac{\dot{H}}{H^2} + 2 \frac{N^{,a} \dot{N}_{,a}}{H N^{,c} N_{,c}}. \quad (20)$$

[‡] It is to be noted that the subsidiary condition is the sufficient condition for slow-roll and the necessary condition is that Eq. (14) multiplied by ϵ, η_{ab} or δ is sufficiently small.

The first term in the right-hand-side of Eq.(20) is given by Eq.(15) and the second term can be simplified using the following relations [9]

$$\dot{N}_{,a} = \dot{\phi}^b \nabla_b \nabla_a N = \nabla_a (\dot{\phi}^b \nabla_b N) - (\nabla_a \dot{\phi}^b) (\nabla_b N) = -H_{,a} - N^{,b} \nabla_a \dot{\phi}_b \quad (21)$$

$$\nabla_a \dot{\phi}_b \simeq -\nabla_a (V_{eff,b}/3H) = \frac{H_{,a} V_{eff,b}}{3H^2} - \frac{\nabla_a \nabla_b V_{eff}}{3H}. \quad (22)$$

Hence Eq.(20) can be written as[§]

$$n_S - 1 = -2\epsilon - \delta - \frac{2\kappa^2}{\Omega N_{,c} N^{,c}} - \frac{2\Omega_{,a} N^{,a}}{\Omega N_{,c} N^{,c}} + \frac{2\eta_{ab} N^{,a} N^{,b}}{N_{,c} N^{,c}}. \quad (23)$$

The expression can be further simplified using Eq.(19) for the case when $N = \text{constant}$ hypersurfaces coincide with $V_{eff} = \text{constant}$ hypersurfaces [10]. In this case \perp_a in Eq.(19) is vanishing, which corresponds to neglecting the isocurvature mode [11]^{||}, and we obtain a very simple formula for the scalar spectral index

$$n_S - 1 = -6\epsilon - 3\delta + 2\eta_{ab} M^{ab}, \quad (24)$$

$$M^{ab} = \frac{N^{,a} N^{,b}}{N_{,c} N^{,c}} = \frac{V_{eff}^{,a} V_{eff}^{,b}}{V_{eff,c} V_{eff}^{,c}} = \frac{1}{\Omega} \widehat{M}^{ab}, \quad (25)$$

which looks quite similar to that for a single field [5]. Moreover, using the relation Eqs.(A.11) and (A.12), one may confirm the conformal invariance of the spectral index

$$n_S - 1 = -6\epsilon - 3\delta + 2\eta_{ab} M^{ab} = -6\widehat{\epsilon} + 2\widehat{\eta}_{ab} \widehat{M}^{ab} = \widehat{n}_S - 1. \quad (26)$$

Tensor perturbations are not changed due to the extension to multiple fields. The tensor power spectrum $P_h(k)$ is given by

$$P_h(k) = 8\kappa^2 \left(\frac{\widehat{H}}{2\pi} \right)^2 = \frac{8\kappa^2}{\Omega} \left(\frac{H}{2\pi} \right)^2. \quad (27)$$

Then the tensor spectral index is given by

$$n_T \equiv \frac{d \ln P_h}{d \ln k} = -2\epsilon = -2\widehat{\epsilon} = \widehat{n}_T, \quad (28)$$

which is of course conformally invariant. The tensor to scalar ratio r is also calculated as

$$r \equiv \frac{P_h}{P_{\mathcal{R}}} = \frac{8\kappa^2}{\Omega N_{,c} N^{,c}} = \frac{8\kappa^2}{\widehat{h}^{ab} \widehat{N}_{,a} \widehat{N}_{,b}} = \widehat{r}, \quad (29)$$

which is again conformally invariant. r can be written using Eq.(19) as

$$r = \frac{16\epsilon}{1 + \frac{2\Omega_{,c}}{\kappa^2} \perp_c \perp^c} \leq 16\epsilon = -8n_T, \quad (30)$$

where we have assumed the positivity of the scalar field metric h_{ab} so that $\perp_c \perp^c \geq 0$. For the case when $N = \text{constant}$ hypersurfaces coincide with $V_{eff} = \text{constant}$ hypersurfaces,

[§] Unlike [8, 9], the term proportional to the Riemann tensor of h_{ab} , which is higher order in slow-roll approximations, does not appear here because the slow-roll equation of motion is used to calculate $\nabla_a \dot{\phi}_b$.

^{||} Note that these modes are initially isocurvature, but they could be adiabatic later like a curvaton and/or a modulated reheating scenario.

	Jordan frame $g_{\mu\nu}$	Einstein frame $\hat{g}_{\mu\nu} = \Omega g_{\mu\nu}$
slow-roll parameters	$\epsilon, \eta_{ab}, \delta$	$\hat{\epsilon} = \epsilon, \hat{\eta}_{ab}$
scalar spectral index n_S	$1 - 6\epsilon - 3\delta + 2\eta_{ab}M^{ab}$	$1 - 6\hat{\epsilon} + 2\hat{\eta}_{ab}M^{ab}$
tensor spectral index n_T	-2ϵ	$-2\hat{\epsilon}$
tensor/scalar ratio r	$16\epsilon = -8n_T$	$16\hat{\epsilon} = -8\hat{n}_T$

Table 1. Slow-roll parameters and inflationary observables in Jordan/Einstein frame for the case when $N = \text{constant}$ hypersurfaces coincide with $V_{eff} = \text{constant}$ hypersurfaces.

we have $r = -8n_T$. However in general due to the presence of isocurvature mode corresponding to \perp_a in Eq.(19) the equality becomes an inequality: $r \leq -8n_T$. These results are summarized in Table 1.

Here, it should be noted that $V_{eff} = \text{constant}$ hypersurfaces coincide with $\hat{V} = \text{constant}$ hypersurfaces. Therefore, the decomposition into adiabatic and isocurvature modes both in the Jordan frame and the Einstein frame also coincide, which, together with the conformal invariance of the curvature perturbation ζ [5, 6], leads to the conclusion that we cannot discriminate the frames by the observations of the primordial fluctuations up to the leading order.

2.3. Example: Extended Assisted Inflation

As an example of multi-scalar extended slow-roll inflation, we consider the assisted inflation model [12, 13]: n scalar fields ϕ^a each with an identical potential $V(\phi^a)$. We assume a flat scalar field metric, $h_{ab} = \delta_{ab}$.

For minimally coupled scalar fields, it was shown that inflation can proceed even if each of the individual fields has a potential too steep to sustain inflation on its own [12, 13]. This can be immediately seen from the action Eq. (1). First, consider a set of n minimally coupled ($\Omega = 1$) scalar fields each with the same potential

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \sum_{a=1}^n \left(\frac{1}{2} (\partial\phi^a)^2 + V(\phi^a) \right) \right]. \quad (31)$$

Since the cross coupling between different fields is absent, the system is equivalent to n copies of the same field and the action is rewritten as

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{n}{2} (\partial\phi^1)^2 - nV(\phi^1) \right] \\ &= \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} R - \frac{1}{2} (\partial\tilde{\phi}^1)^2 - \tilde{V}(\tilde{\phi}^1) \right], \end{aligned} \quad (32)$$

where

$$\tilde{\phi}^1 = \sqrt{n}\phi^1, \quad \tilde{V} = nV. \quad (33)$$

Thus, the system is equivalent to a single scalar field with a potential of the same form. For example, for an exponential potential, $V(\phi) = V_0 \exp(-\lambda\kappa\phi)$, the redefined potential

becomes,

$$\tilde{V}(\tilde{\phi}) = nV_0 \exp\left(-(\lambda/\sqrt{n})\kappa\tilde{\phi}\right) =: \tilde{V}_0 \exp(-\tilde{\lambda}\kappa\tilde{\phi}). \quad (34)$$

Hence for large n the slope of the potential $\tilde{V}(\tilde{\phi})$ becomes less steep than that of $V(\phi)$ [12]. In fact, the slow-roll parameters are given by $\tilde{\epsilon} = \tilde{\lambda}^2/2 = \lambda^2/(2n) = \epsilon_1/n$ and $\tilde{\eta} = \tilde{\lambda}^2 = \lambda^2/n = \eta_1/n$, where ϵ_1 and η_1 are the slow-roll parameters of a single field, and hence are suppressed by n . Then, the observable quantities are given by $n_S - 1 = n_T = -r/8 = -\lambda^2/n$. For a monomial potential, $V(\phi) = \lambda\phi^p$, the redefined potential becomes,

$$\tilde{V}(\tilde{\phi}) = (\lambda/n^{p/2-1})\tilde{\phi}^p =: \tilde{\lambda}\tilde{\phi}^p, \quad (35)$$

and for large n the self-interactions become weaker for $p > 2$ [13]. In this case, however, the slow-roll parameters are not changed and the observable quantities are given by $n_S - 1 = -p(p+2)/(\kappa^2\tilde{\phi}^2) = -(p+2)/(2N)$ and $n_T = -r/8 = -p^2/(\kappa^2\tilde{\phi}^2) = -p/(2N)$, with N being the e-folding number until the end of inflation.

Even if a non-minimal coupling is introduced, assisted inflation can persist and a redefinition Eq. (33) again helps to flatten the potential but does not help to weaken the non-minimally coupling. To be specific, consider the n copies of non-minimally coupled with a potential with $\Omega(\phi) = 1 - \xi\kappa^2 \sum_{a=1}^n (\phi^a)^2 = \frac{1}{n} \sum_{a=1}^n \tilde{\Omega}(\phi^a)$ with $\tilde{\Omega}(\phi^a) = 1 - n\xi\kappa^2(\phi^a)^2$

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \left[\frac{\Omega(\phi)}{2\kappa^2} R - \sum_{a=1}^n \left(\frac{1}{2}(\partial\phi^a)^2 + V(\phi^a) \right) \right] \\ &= \int d^4x \sqrt{-g} \left[\sum_{a=1}^n \left(\frac{\tilde{\Omega}(\phi^a)}{2n\kappa^2} R - \frac{1}{2}(\partial\phi^a)^2 - V(\phi^a) \right) \right] \\ &= \int d^4x \sqrt{-g} \left[\frac{\tilde{\Omega}(\phi^1)}{2\kappa^2} R - \frac{n}{2}(\partial\phi^1)^2 - nV(\phi^1) \right]. \end{aligned} \quad (36)$$

Thus, the system is equivalent to a non-minimally coupled single field with the conformal factor and the potential of the same form. Similar to the minimally coupled case, the field redefinition Eq. (33) does help to flatten the potential as in Eq. (34) and Eq. (35), but the non-minimal coupling parameters ξ is the same: $\tilde{\Omega}(\tilde{\phi}) = 1 - n\xi\kappa^2\tilde{\phi}^2 = 1 - \xi\kappa^2\tilde{\phi}^2$. For example, for $V(\phi) = \lambda\phi^p$, observational quantities become [5] $n_S - 1 = n_T = -r/8 \simeq (p-4)^2\xi$, for $|\xi|\kappa^2\tilde{\phi}^2 \gg 1$ and $\xi < 0$, if $|\xi| \ll 1$ and $p \neq 4$. Because of the invariance of ξ , the slow-roll parameters and n_S, n_T and r are not changed in this case.

3. Slow-Roll Inflation with Generalized Gravity

The action we consider is [14]

$$S = \int d^4x \sqrt{-g} \left[\frac{f(\phi, R)}{2\kappa^2} - \frac{1}{2}\omega(\phi)(\nabla\phi)^2 - V(\phi) \right]. \quad (37)$$

Firstly, we show that the action is equivalent to that of two scalar fields non-minimally coupled to gravity with degenerate scalar field metric h_{ab} . By introducing auxiliary field ψ , Eq.(37) is dynamically equivalent to [15]

$$S = \int d^4x \sqrt{-g} \left[\frac{f(\phi, \psi)}{2\kappa^2} + \frac{f_{,\psi}}{2\kappa^2}(R - \psi) - \frac{1}{2}\omega(\phi)(\nabla\phi)^2 - V(\phi) \right], \quad (38)$$

where $f_{,\psi} = \partial f / \partial \psi$. One can easily verify that the equation of motion for ψ gives $\psi = R$ if $f_{,\psi\psi} \neq 0$, which reproduces the original action. Eq.(38) is the action of two scalar fields non-minimally coupled to gravity ($\Omega = f_{,\psi}$) with the unusual metric h_{ab} : $h_{\phi\phi} = \omega$, $h_{\psi\psi} = 0$. Thus, the generalized gravity theory is equivalent to two scalar fields theory non-minimally coupled to gravity [16, 17].

Assuming that the universe is described by the flat, homogeneous, and isotropic universe model with the scale factor a , the field equations derived from Eq.(38) are given by

$$H^2\Omega + H\dot{\Omega} = \frac{\kappa^2}{3} \left[\frac{1}{2}\omega\dot{\phi}^2 + V + \frac{1}{2\kappa^2}(\psi\Omega - f) \right] =: \frac{\kappa^2}{3} \left(\frac{1}{2}\omega\dot{\phi}^2 + U \right), \quad (39)$$

$$\ddot{\Omega} - H\dot{\Omega} + 2\dot{H}\Omega = -\kappa^2\omega\dot{\phi}^2, \quad (40)$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{1}{\omega} \left(\frac{1}{2}\omega_{,\phi}\dot{\phi}^2 + V_{,\phi} - \frac{f_{,\phi}}{2\kappa^2} \right) = -\frac{1}{\omega} \left(\frac{1}{2}\omega_{,\phi}\dot{\phi}^2 + U_{,\phi} - \frac{\psi\Omega_{,\phi}}{2\kappa^2} \right) \quad (41)$$

where $\Omega = f_{,\psi}$ is the conformal factor.

3.1. Slow-Roll Conditions

As done in the previous section, we assume that the time scale of the motion of the scalar field is much larger than the cosmic time scale H^{-1} under the slow-roll approximations. Then, as slow-roll conditions of the scalar field in the generalized gravity, we impose that $\omega\dot{\phi}^2 \ll |U|$, $|\dot{\Omega}| \ll H\Omega$, $|\ddot{\phi} + \omega_{,\phi}\dot{\phi}^2/(2\omega)| \ll H|\dot{\phi}|$, $|\ddot{\phi} + \omega_{,\phi}\dot{\phi}^2/(2\omega)| \ll |U_{,\phi}/\omega|$, which yields

$$H^2\Omega \simeq \frac{\kappa^2}{3}U, \quad (42)$$

$$3H\dot{\phi} \simeq -\frac{U_{,\phi}}{\omega} + 6\frac{\Omega_{,\phi}H^2}{\kappa^2\omega} \simeq -\frac{\Omega^2}{\omega} \left(\frac{U}{\Omega^2} \right)_{,\phi} =: -V_{,\phi}^{eff}, \quad (43)$$

where in the second equation, we have assumed $|\dot{H}/H^2| \ll 1$, which is necessary to cause inflation and should be checked later. This amounts to assuming that only ϕ becomes an inflaton and ψ is a dependent variable of ϕ .[¶]

In the following, we derive the consistency conditions for the slow-rolling of the scalar field in the generalized gravity (the generalized slow-roll conditions). By

[¶] To be more detailed, we may regard Eq. (42) as the equation of the variables ϕ and ψ , that is, $\psi \simeq 4\kappa^2 U(\phi, \psi)$ and solve it implicitly to express $\psi = \psi(\phi)$ [18]. Then, by inserting this expression ψ into the original potential $U(\phi, \psi)$ and the conformal factor $\Omega(\phi, \psi)$, we obtain the reduced potential $U(\phi, \psi(\phi))$ and the reduced conformal factor $\Omega(\phi, \psi(\phi))$.

computing $\ddot{\phi} + \omega_\phi \dot{\phi}^2/(2\omega)$ from Eq. (43), we obtain

$$\frac{\ddot{\phi} + \omega_\phi \dot{\phi}^2/(2\omega)}{H\dot{\phi}} \simeq -\frac{\dot{H}}{H^2} \left(1 + \frac{8\kappa^2 U V_{,\phi\psi}^{eff}}{\Omega V_{,\phi}^{eff}} \right) - \frac{\Omega}{\kappa^2 U} \left(V_{,\phi\phi}^{eff} + \frac{\omega_{,\phi}}{2\omega} V_{,\phi}^{eff} \right), \quad (44)$$

where we have used $\dot{\psi} = \dot{R} \simeq 24H\dot{H}$. Also, from Eq. (43), we note

$$\frac{\ddot{\phi} + \omega_\phi \dot{\phi}^2/(2\omega)}{U_{,\phi}} \simeq -\frac{\ddot{\phi} + \omega_\phi \dot{\phi}^2/(2\omega)}{3H\dot{\phi}} \frac{V_{,\phi}^{eff}}{U_{,\phi}}. \quad (45)$$

Moreover, from Eqs. (42) and (43),

$$\left| \frac{\omega \dot{\phi}^2}{U} \right| \simeq \frac{\omega \Omega (V_{,\phi}^{eff})^2}{3\kappa^2 U^2}, \quad (46)$$

$$\frac{\dot{\Omega}}{H\Omega} \simeq -\frac{\Omega_{,\phi} V_{,\phi}^{eff}}{\kappa^2 U} + \frac{2\Omega_{,\psi} \psi}{\Omega} \frac{\dot{H}}{H^2}. \quad (47)$$

Hence, we introduce the following three slow-roll parameters and obtain the generalized slow-roll conditions:

$$\epsilon^g := \frac{\omega \Omega (V_{,\phi}^{eff})^2}{2\kappa^2 U^2}; \quad \epsilon^g \ll 1, \quad (48)$$

$$\eta^g := \frac{\Omega}{\kappa^2 U} \left(V_{,\phi\phi}^{eff} + \frac{\omega_{,\phi}}{2\omega} V_{,\phi}^{eff} \right); \quad |\eta^g| \ll 1, \quad (49)$$

$$\delta^g := \frac{\Omega_{,\phi} V_{,\phi}^{eff}}{\kappa^2 U}; \quad |\delta^g| \ll 1, \quad (50)$$

and three subsidiary conditions:

$$\left| \frac{\kappa^2 U V_{,\phi\psi}^{eff}}{\Omega V_{,\phi}^{eff}} \right| = \mathcal{O}(1), \quad (51)$$

$$\left| \frac{\kappa^2 \Omega_{,\psi} U}{\Omega^2} \right| = \mathcal{O}(1), \quad (52)$$

$$\left| \frac{U_{,\phi}}{\omega V_{,\phi}^{eff}} \right| = \mathcal{O}(1). \quad (53)$$

Note that since from Eq. (42) \dot{H}/H^2 is approximated as

$$\frac{\dot{H}}{H^2} \simeq -\frac{\epsilon^g + \frac{1}{2}\delta^g}{1 - \frac{\Omega_{,\psi}\psi}{\Omega}}. \quad (54)$$

Thus, $|\dot{H}/H^2| \ll 1$ is guaranteed by these slow-roll conditions unless $1 - \Omega_{,\psi}\psi/\Omega$ becomes vanishing accidentally.⁺

⁺ It is interesting to note that $\Omega_{,\psi}\psi = \Omega$ implies $f(\phi, R) = f(\phi)R^2 + g(\phi)$ with $f(\phi)$ and $g(\phi)$ being arbitrary functions of ϕ , which corresponds to R^2 inflation model [17]. In this case, both ϕ and ψ become inflatons and higher order corrections of slow-roll approximations become important. However, in the case when $f(\phi, \psi) = f(\psi)$, only ψ can be an inflaton and the primordial fluctuations can be calculated in the Einstein frame as given in Appendix A.2.2.

To sum up, the generalized slow-roll conditions consist of three conditions Eqs. (48), (49), (50) and three subsidiary condition Eqs. (51), (52), and (53).*

As an example, we consider the case of a non-minimal coupling $f = R\Omega(\phi)$. In this case, $U(\phi, \psi) = V(\phi)$ and $V^{eff}(\phi, \psi) = V^{eff}(\phi)$. Then, all slow-roll conditions coincide with those in the previous section.

3.2. Perturbations

In this subsection, we would like to evaluate the primordial density fluctuations generated in the generalized gravity theory. Since only ϕ is an independent field to the first order in the slow-roll approximations, the system is equivalent to one scalar field (ϕ) non-minimally coupled to gravity, which has been already studied [5].

From the conformal invariance of the curvature perturbation \mathcal{R} , the power spectrum $P_{\mathcal{R}}(k)$ in the Jordan frame is given by that in the Einstein frame [7, 8, 9, 6],

$$P_{\mathcal{R}}(k) = \left(\frac{\hat{H}}{2\pi} \right)^2 \hat{N}_{,\phi}^2 = \left(\frac{H}{2\pi} \right)^2 \frac{1}{\omega} N_{,\phi}^2, \quad (55)$$

where k is a comoving wavenumber at the horizon exit ($k = aH$) and the hatted variables are those in the Einstein frame defined in Appendix A and we have used $d\hat{\phi} = d\phi\sqrt{\omega/\Omega}$ and $\hat{H} \simeq H/\sqrt{\Omega}$ in the slow-roll approximation. $N(\phi)$ is conformally invariant in the slow-roll approximation, $\hat{N} = N$. Then, using the relation Eqs.(A.22) and (A.23), the spectral index of scalar perturbation is given by

$$n_S - 1 = -6\epsilon^g + 2\eta^g - 3\delta^g = -6\hat{\epsilon}^g + 2\hat{\eta}^g = \hat{n}_S - 1, \quad (56)$$

which proves the conformal invariance of the spectral index [5]. The tensor spectral index n_T and the tensor to scalar ratio r are also conformal invariant [5].

4. Summary

By extending our previous study, we have derived the slow-roll conditions for multiple scalar fields non-minimally coupled to gravity. The slow-roll conditions consist of three main conditions Eqs. (11), (12), (13) and one subsidiary condition Eq. (14). We have given the simple formulae of spectral indices and the tensor-to scalar ratio, Eq. (23), Eq. (28) and Eq. (29). The scalar spectral index can be further simplified and be written in terms of the slow-roll parameters if the isocurvature perturbation is negligible, Eq. (24).

We have also derived the slow-roll conditions for generalized gravity theories and found that, to the first order in the slow-roll approximations, the system is reduced to a single field with non-minimal coupling. The formulae of the spectral indices are thus the same as our previous results [5].

* It should be noted that three subsidiary conditions are the sufficient conditions for slow-roll and the necessary conditions are that Eq. (51) (or Eq. (52) or Eq. (53)) multiplied by \dot{H}/H^2 are sufficiently small.

We hope that our formulae may be useful to connect the inner space to the outer space.

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Appendix A. Slow-Roll Conditions and Perturbations in Einstein Frame

In this appendix, we perform the conformal transformation to the Einstein frame and introduce slow-roll parameters and give their relations with those in the Jordan frame.

Appendix A.1. A Non-minimally Coupled Multi-Scalar Field

First, we consider the case of non-minimally coupled multiple scalar fields. Introducing the Einstein metric $\hat{g}_{\mu\nu}$ by the conformal transformation

$$\hat{g}_{\mu\nu} = \Omega(\phi)g_{\mu\nu}, \quad (\text{A.1})$$

the action Eq.(1) becomes that of two scalar fields minimally coupled to Einstein gravity

$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} \hat{R} - \frac{1}{2\Omega} \left(h_{ab} + \frac{3\Omega_{,a}\Omega_{,b}}{2\kappa^2\Omega} \right) \hat{g}^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - \frac{V}{\Omega^2} \right] \quad (\text{A.2})$$

$$= \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa^2} \hat{R} - \frac{1}{2} \hat{h}_{ab} \hat{g}^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - \hat{V}(\phi) \right], \quad (\text{A.3})$$

where \hat{h}_{ab} and \hat{V} are defined by

$$\hat{V}(\phi) = \frac{V(\phi)}{\Omega(\phi)^2}, \quad (\text{A.4})$$

$$\hat{h}_{ab} = \frac{1}{\Omega} \left(h_{ab} + \frac{3\Omega_{,a}\Omega_{,b}}{2\kappa^2\Omega} \right). \quad (\text{A.5})$$

Here, it should be noted that one cannot define $\hat{\phi}^a$ by $\left(d\hat{\phi}^a \right)^2 \equiv (d\phi^a)^2 / \Omega(\phi^a)$ because $dd\hat{\phi}^a \neq 0$, different from a single field case.

Appendix A.1.1. Slow-Roll Conditions The slow-roll conditions in the Einstein frame are simply given by

$$\hat{\epsilon} := \frac{1}{2\kappa^2 \hat{V}^2} \hat{h}^{ab} \hat{V}_{,a} \hat{V}_{,b}; \quad \hat{\epsilon} \ll 1, \quad (\text{A.6})$$

$$\hat{\eta}_{ab} := \frac{\hat{\nabla}_a \hat{\nabla}_b \hat{V}}{\kappa^2 \hat{V}}; \quad |\hat{\eta}_a^b| \ll 1. \quad (\text{A.7})$$

$$(\text{A.8})$$

Using Eq. (A.4) and Eq. (6), these parameters are rewritten as

$$\widehat{\epsilon} = \frac{\widehat{h}^{ab} V_{eff,a} V_{eff,b}}{2\kappa^2 V^2}, \quad (\text{A.9})$$

$$\widehat{\eta}_{ab} = \frac{\Omega^2}{\kappa^2 V} \widehat{\nabla}_a \left(\frac{\nabla_b V_{eff}}{\Omega^2} \right), \quad (\text{A.10})$$

where V_{eff}^a is defined in Eq. (6). If we assign $\mathcal{O}(\varepsilon/\sqrt{n})$ to $\sqrt{\Omega} V_{eff,a}/\kappa V$ and to $\Omega_{,a}/\kappa\sqrt{\Omega}$, then from Eq.(11) and Eq.(13), we have $\epsilon = \mathcal{O}(\varepsilon^2)$ and $\delta = \mathcal{O}(\varepsilon^2)$. Therefore, under the slow-roll conditions in the Jordan frame, $\Omega_{,a}\Omega_{,b}/\kappa^2\Omega = \mathcal{O}(\varepsilon^2/n)$ and $\widehat{h}_{ab} \simeq h_{ab}/\Omega$ is implied. Hence the slow-roll parameters in the Einstein frame are related to the slow-roll parameters in the Jordan frame as

$$\widehat{\epsilon} \simeq \epsilon, \quad (\text{A.11})$$

$$\widehat{\eta}_{ab} \simeq \frac{1}{\Omega} \left(\eta_{ab} - \frac{1}{2} h_{ab} \delta - \frac{3\Omega_{,a} V_{eff,b}}{2\kappa^2 V} + \frac{\Omega_{,b} V_{eff,a}}{2\kappa^2 V} \right), \quad (\text{A.12})$$

where we have used the relation [19]

$$\widehat{\nabla}_a v_b = \nabla_a v_b - C_{ab}^c v_c, \quad (\text{A.13})$$

$$C_{ab}^c = \frac{1}{2} (h_{ab} \nabla^c \ln \Omega - \delta_a^c \nabla_b \ln \Omega - \delta_b^c \nabla_a \ln \Omega). \quad (\text{A.14})$$

Appendix A.2. Generalized Gravity

Next, we consider the case of the generalized gravity with a single scalar field Eq.(37). From the equivalent action Eq.(38), introducing the Einstein metric $\widehat{g}_{\mu\nu}$ by the conformal transformation

$$\widehat{g}_{\mu\nu} = f_{,\psi}(\phi, \psi) g_{\mu\nu} \equiv \Omega(\phi, \psi) g_{\mu\nu}, \quad (\text{A.15})$$

the equivalent action Eq.(38) becomes that of two scalar fields minimally coupled to Einstein gravity

$$S = \int d^4x \sqrt{-\widehat{g}} \left[\frac{1}{2\kappa^2} \widehat{R} - \frac{\omega}{2\Omega} (\widehat{\nabla}\phi)^2 - \frac{3}{4\kappa^2\Omega^2} (\widehat{\nabla}\Omega)^2 - \frac{V}{\Omega^2} - \frac{\psi\Omega - f}{2\kappa^2\Omega^2} \right] \quad (\text{A.16})$$

Appendix A.2.1. Slow-Roll Conditions Only ϕ can be an inflaton (see Eq. (43)) and $\psi \simeq 12H^2$ is a dependent variable of ϕ , to the first order in slow-roll approximations.‡ In the case when $\psi = \psi(\phi)$, the action becomes

$$\begin{aligned} S &= \int d^4x \sqrt{-\widehat{g}} \left[\frac{1}{2\kappa^2} \widehat{R} - \frac{\omega}{2\Omega} \left(1 + \frac{3\Omega_{,\phi}^2}{2\kappa^2\omega\Omega} \right) (\widehat{\nabla}\phi)^2 - \frac{U}{\Omega^2} \right] \\ &= \int d^4x \sqrt{-\widehat{g}} \left[\frac{1}{2\kappa^2} \widehat{R} - \frac{1}{2} (\widehat{\nabla}\widehat{\phi})^2 - \widehat{V}(\widehat{\phi}) \right], \end{aligned} \quad (\text{A.17})$$

‡ Note that from the slow-roll equation of motion, $\psi \simeq 12H^2 \simeq 4\kappa^2 U/\Omega$, $\widehat{V}_{,\psi} \simeq 0$ under the slow-roll approximation. So ψ stays the minimum if it is massive, being consistent with our assumption. We do not consider the case when ψ is light so that both ϕ and ψ can be inflatons. For the case when only ψ can be an inflaton, see below.

where U is defined by Eq. (39) and we have introduced a canonically normalized scalar field $\widehat{\phi}$ with a potential $\widehat{V}(\widehat{\phi})$

$$(d\widehat{\phi})^2 = \frac{\omega}{\Omega} \left(1 + \frac{3\Omega_{,\phi}^2}{2\kappa^2\omega\Omega} \right) (d\phi)^2, \quad (\text{A.18})$$

$$\widehat{V} = \frac{U}{\Omega^2} = \frac{1}{\Omega^2} \left(V + \frac{\psi\Omega - f}{2\kappa^2} \right). \quad (\text{A.19})$$

The slow-roll parameters and conditions in the Einstein frame are simply

$$\widehat{\epsilon}_g := \frac{(\widehat{V}_{,\widehat{\phi}})^2}{2\kappa^2\widehat{V}^2}; \quad \widehat{\epsilon}_g \ll 1, \quad (\text{A.20})$$

$$\widehat{\eta}_g := \frac{\widehat{V}_{,\widehat{\phi}\widehat{\phi}}}{\kappa^2\widehat{V}}; \quad |\widehat{\eta}_g| \ll 1. \quad (\text{A.21})$$

If we assign $\mathcal{O}(\varepsilon^2)$ to the slow-roll parameters $(\epsilon_g, \eta_g, \delta_g)$, then $3\Omega_{,\phi}^2/(2\kappa^2\omega\Omega) = \mathcal{O}(\varepsilon^2)$, and $d\widehat{\phi}^2 \simeq \omega d\phi^2/\Omega$ is satisfied under the slow-roll conditions Eq.(48) and Eq.(50). Therefore, under the slow-roll approximations, the slow-roll parameters in the Einstein frame are related to those in the Jordan frame as [5]

$$\widehat{\epsilon}_g \simeq \epsilon_g, \quad (\text{A.22})$$

$$\widehat{\eta}_g \simeq \eta - \frac{3}{2}\delta. \quad (\text{A.23})$$

Appendix A.2.2. $f(R)$ Inflation Different from the previous subsection, we consider the case that only ψ exists and is an inflaton, that is, $f(\phi, \psi) = f(\psi)$ and $\omega(\phi) = V(\phi) = 0$. Then, the equivalent action Eq.(38) in the Einstein frame becomes

$$\begin{aligned} S &= \int d^4x \sqrt{-\widehat{g}} \left[\frac{1}{2\kappa^2} \widehat{R} - \frac{3\Omega_{,\psi}^2}{4\kappa^2\Omega^2} (\widehat{\nabla}\psi)^2 - \frac{\psi\Omega - f}{2\kappa^2\Omega^2} \right] \\ &= \int d^4x \sqrt{-\widehat{g}} \left[\frac{1}{2\kappa^2} \widehat{R} - \frac{1}{2} (\widehat{\nabla}\widehat{\psi})^2 - \widehat{V}(\widehat{\psi}) \right], \end{aligned} \quad (\text{A.24})$$

where $(d\widehat{\psi})^2 = 3\Omega_{,\psi}^2(d\psi)^2/(2\kappa^2\Omega^2)$ and $\widehat{V}(\widehat{\psi}) = (\psi\Omega - f)/(2\kappa^2\Omega^2)$. Then, the slow-roll parameters and conditions are given by

$$\widehat{\epsilon}_g := \frac{(\widehat{V}_{,\widehat{\psi}})^2}{2\kappa^2\widehat{V}^2}; \quad \widehat{\epsilon}_g \ll 1, \quad (\text{A.25})$$

$$\widehat{\eta}_g := \frac{\widehat{V}_{,\widehat{\psi}\widehat{\psi}}}{\kappa^2\widehat{V}}; \quad |\widehat{\eta}_g| \ll 1. \quad (\text{A.26})$$

Here $\widehat{V}_{,\widehat{\psi}}$ and $\widehat{V}_{,\widehat{\psi}\widehat{\psi}}$ are given by

$$\widehat{V}_{,\widehat{\psi}} = \frac{1}{\sqrt{6}\kappa\Omega} \left(-\psi + \frac{2f}{\Omega} \right), \quad (\text{A.27})$$

$$\widehat{V}_{,\widehat{\psi}\widehat{\psi}} = \frac{1}{3} \left[\frac{2}{\Omega} \left(\psi - \frac{2f}{\Omega} \right) + \frac{1}{\Omega_{,\psi}} \left(1 - \frac{\psi\Omega_{,\psi}}{\Omega} \right) \right]. \quad (\text{A.28})$$

It is to be noted that the condition $\widehat{V}_{,\widehat{\psi}} = 0$ is equivalent to $f \propto \psi^2$. Furthermore, one can easily check that $\widehat{V}_{,\widehat{\psi}\widehat{\psi}} = 0$ for $f \propto \psi^2$. Thus, as is well known, only the models with $f(\psi)$ which slightly deviates from ψ^2 can cause inflation.

As examples, we consider two models. The first one is a power law model: $f(\psi) = A\psi^{2+\alpha}$ ($\alpha \ll 1, A$: constants). Then, the slow-roll parameters are given by

$$\widehat{\epsilon}_g = \frac{\alpha^2}{3(1+\alpha)^2}, \quad (\text{A.29})$$

$$\widehat{\eta}_g = \frac{2\alpha^2}{3(1+\alpha)^2}. \quad (\text{A.30})$$

Then, the observable quantities are given by

$$\widehat{n}_S - 1 = -6\widehat{\epsilon}_g + 2\widehat{\eta}_g = -\frac{2\alpha^2}{3(1+\alpha)^2}, \quad (\text{A.31})$$

$$\widehat{n}_T = -\frac{\widehat{r}}{8} = -2\widehat{\epsilon}_g = -\frac{2\alpha^2}{3(1+\alpha)^2}. \quad (\text{A.32})$$

As the second model we consider the Starobinsky model [20]: $f(\psi) = \psi + \psi^2/(6M^2)$ ($M \neq 0$: constant). In terms of $\kappa\widehat{\psi} = \sqrt{\frac{3}{2}} \ln \Omega$, the potential \widehat{V} becomes

$$\widehat{V} = \frac{\psi^2}{12\kappa^2 M^2 \left(1 + \frac{\psi}{3M^2}\right)^2} = \frac{3M^2}{4\kappa^2} \left(1 - e^{-\sqrt{\frac{2}{3}}\kappa\widehat{\psi}}\right)^2. \quad (\text{A.33})$$

The slow-roll parameters are then given by

$$\widehat{\epsilon}_g = \frac{12M^4}{\psi^2} = \frac{4}{3 \left(e^{\sqrt{\frac{2}{3}}\kappa\widehat{\psi}} - 1\right)^2} \simeq \frac{3}{4N^2}, \quad (\text{A.34})$$

$$\widehat{\eta}_g = \frac{4M^2(3M^2 - \psi)}{\psi^2} = \frac{4 \left(2 - e^{\sqrt{\frac{2}{3}}\kappa\widehat{\psi}}\right)}{3 \left(e^{\sqrt{\frac{2}{3}}\kappa\widehat{\psi}} - 1\right)^2} \simeq -\frac{1}{N}, \quad (\text{A.35})$$

where the e-folding number N is given by $N \simeq \psi/(4M^2) \simeq 3e^{\sqrt{\frac{2}{3}}\kappa\widehat{\psi}}/4$. Then, the observable quantities are given by

$$\widehat{n}_S - 1 = -6\widehat{\epsilon}_g + 2\widehat{\eta}_g \simeq -\frac{9}{2N^2} - \frac{2}{N}, \quad (\text{A.36})$$

$$\widehat{n}_T = -\frac{\widehat{r}}{8} = -2\widehat{\epsilon}_g = -\frac{3}{2N^2}, \quad (\text{A.37})$$

in agreement with [21].

As a final remark, it should be noted that using the slow-roll equation of motion, $\psi \simeq 12H^2 \simeq 12\widehat{H}^2\Omega \simeq 4\kappa^2\Omega\widehat{V}$, we find $\widehat{V}_{,\widehat{\psi}} \simeq 0$ up to the leading order of the slow-roll approximations. That is, in order to relate these results in the Einstein frame to those in the Jordan frame, one needs to go beyond the leading order of slow-roll approximations.

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